# A brief remark on Unruh effect and causality

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## Abstract

Unruh effect states that the vacuum of a quantum field theory on Minkovski space-time looks like a thermal state for an eternal uniformly accelerated observer. Adaptation to the non eternal case causes a serious problem: if the thermalization of the vacuum depends on the lifetime of the observer, then in principle the latest is able to deduce its lifetime from the measurement of the temperature. This short note aims at underlining that time-energy uncertainty relation allows to adapt Unruh effect to non-eternal observers without breaking causality. In particular we show that our adaptation - the *diamonds's temperature*- of Bisognano-Wichman approach to Unruh effect is causally acceptable. This note is self-contained but it is fully meaningful as a complement to gr-qc/0212074 as well as a comment on gr-qc/0306022.

## I Introduction

The Unruh effect[16] states that an observer in Minkovski spacetime $M$ with constant acceleration $a$ and *infinite lifetime* sees the vacuum of a quantum field theory on $M$ as a thermal equilibrium state with temperature

$$T_U = \frac{\hbar a}{2\pi k_b c}. \tag{1}$$

This result can be obtained by observing that the vacuum for a quantization scheme on all $M$ is not a pure state for an alternative (but as well defined) quantization prescription on the Rindler wedge $W$. The latest is physically relevant for $W$ is the (whole and only) region of $M$ with whom an eternal uniformly accelerated observer can interact, i.e. send a request and obtain an answer. For a non-eternal observer $W$ has no particular signification and the comparison between the two quantization prescriptions is no longer significant. In this sense the eternity of the observer is a strong requirement to derive Unruh's result. Since no eternal observer exists this gives an equally strong argument to question the validity of the effect[6]. Of course this objection may be overcome by viewing $T_U$ as a limit for asymptotic states[10,12] but such a limit is not always meaningful. For instance $T_U$ may be interpreted in terms of Hawking radiation[2] for eternal black holes but not for Kerr black holes because[17] the Killing field generating the horizon in Kerr spacetime has spacelike orbits near infinity.

Several adaptations of $T_U$ have been proposed[11,13,14] for an observer with a finite lifetime $\mathcal{T}$.* If the thermalization of the vacuum survives in the non-eternal case then either the temperature $T$ does not depend on the lifetime and coherence with the eternal case yields

$$T = T_U \tag{2}$$

for all $\mathcal{T}$, or

$$T = T(\mathcal{T}) \tag{3}$$

with

$$\lim_{\mathcal{T}\to+\infty} T(\mathcal{T}) = T_U. \tag{4}$$

In this last case we face a severe problem: by measuring a temperature $T_0$ an observer knowing (3) would be able to deduce his lifetime $T^{-1}(T_0)$ and so he could predict the instant of his death. How then can the temperature depend on the lifetime without breaking causality ?

This note aims at underlining that time-energy uncertainty relation prevents (3) from being automatically ruled out by causal considerations. In particular we show that our adaptation[11] of Bisognano-Wichman's[1,15] approach to Unruh effect yields the highest lifetime-depending temperature authorized by uncertainty relation.

## II Time-energy uncertainty and measurement of temperature

The time-energy uncertainty relation[3] states that the time $\Delta t$ needed for a non-dissipative quantum system to evolve in a significant manner is as long as the uncertainty $\Delta E$ on the energy is small,

$$\Delta t \Delta E \geq h. \tag{5}$$

Detectors considered to measure Unruh temperature most often consist in a quantum system $S$ coupled to the vacuum. To measure a temperature $T$ the Unruh detector $S$ should evolve from an initial (ground) state $E$ to an excited (thermal) state $E + k_b T$. The energy gap $k_b T$ can be distinguished from the ground energy level only if the latest is known with accuracy $\Delta E < k_b T$. Consequently a significant evolution of $S$ (e.g. from the ground to an excited state) requires a period of time not shorter than $\frac{h}{k_b T}$. Therefore an observer with lifetime $\mathcal{T}$ is not able to

*$\mathcal{T}$ is measured in the observer's own referential

measure a temperature with accuracy greater than $\frac{h}{k_b\mathcal{T}}$. In particular if

$$T(\mathcal{T}) < \frac{h}{k_b\mathcal{T}} \tag{6}$$

the Unruh observer has no time to measure $T$ precisely enough so that to predict his lifetime. From this point of view a $\mathcal{T}$-dependent temperature satisfying (6) is causally acceptable.

This is not a necessary condition. One may expect $T(\mathcal{T})$ to be obtained from correction of $T_U$ in powers of $\mathcal{T}^{-1}$. If the accuracy in temperature measurement is less than

$$\Delta T(\mathcal{T}) \doteq |T(\mathcal{T}) - T_U| \tag{7}$$

then the observer is not able to affirm that what he is measuring is distinct from what he would measure if he was eternal. In other words he doesn't know whether he may live forever or not. Hence

$$\Delta T(\mathcal{T}) < \frac{h}{k_b\mathcal{T}} \tag{8}$$

is another condition making a $\mathcal{T}$-dependent temperature causally acceptable.

Mathematically one may have (8) with or without (6) and vice versa. Physically what is meaningful is first to check (6) (does the observer have enough time to distinguish $k_bT$ from the ground energy ?) and, in case the answer is no, check (8) (can the observer distinguish $T$ from $T_U$ ?).

Before applying the procedure above to the diamonds's temperature, let us discuss the interpretation of time-energy uncertainty relation. Strictly speaking (5) is valid for a system described by a wave packet and $(\Delta T)^{-1}$ is the frequency of oscillation of the probability $P(b_m, t)$ of obtaining the eigenvalue $b_m$ in the measurement of a given observable $B$ (not commuting with the Hamiltonian). Whether or not a similar interpretation is valid for the measurement of the energy during a transition between ground and excited states is not clear to the author (but this is certainly clarified in the suitable literature). Moreover (5) is valid for non dissipative system, which put some constraint on the accelerating process of the Unruh observer. Both restrictions can be overcome by the following considerations: a quantum system $S$ coupled to the vacuum does not constitute by itself an Unruh thermometer; one also needs a process to measure the energy levels of $S$ (in the same manner that a column of mercury alone is not a thermometer; it requires gradations marks to be readable). To measure energy gap between quantum levels, one applies some time-dependent perturbations in order to localize the resonances of the system. A second version of the time-energy relation[3] indicates that a sinusoidal perturbation acting for a time $\mathcal{T}$, cannot determine resonance with accuracy greater than $\frac{\hbar}{\mathcal{T}}$. Hence conditions similar to (6) and (8) with $h$ replaced by $\hbar$,

$$\begin{aligned} T(\mathcal{T}) &< \frac{\hbar}{k_b\mathcal{T}}, && (9)\\ \Delta T(\mathcal{T}) &< \frac{\hbar}{k_b\mathcal{T}}. && (10)\end{aligned}$$

Since $\hbar < h$, conditions (9) and (10) are stronger than (6) and (8).

# III Diamonds's temperature

Our adaptation of Unruh effect to bounded trajectories[11] is obtained by considering the modular group[8] associated to the region causally connected to an non eternal observer. Concretely $W$ is replaced by a diamond shape region $D \subset M$. Up to an acceptation of KMS conditions[7] as a local definition of a thermal state, the identification of the modular flow to the thermal flow (the thermal time hypothesis[4]) indicates that the vacuum as seen by an observer with lifetime[†]

$$\mathcal{T} = 2\tau_0 \tag{11}$$

is a thermal state whose temperature depends on both $\tau_0$ and the observer proper time $\tau$,

$$T(\tau_0, \tau) = T_U \frac{\sinh a\tau_0}{\cosh a\tau_0 - \cosh a\tau} \tag{12}$$

where we take $c = 1$. The local interpretation of KMS theory is justified a posteriori by noting that for given $\tau_0$ and acceleration $a$, the temperature is almost a constant for most of the lifetime and takes the value observed in the middle of the observer's life,

$$T(\tau_0, \tau) \backsimeq T(\tau_0, 0). \tag{13}$$

(13) is called the *diamond's* temperature

$$T_D(a, \tau_0) \doteq T_U \frac{\cosh a\tau_0 + 1}{\sinh a\tau_0}. \tag{14}$$

With respect to condition (9) diamonds's temperature is causally acceptable for small accelerations

$$\lim_{a\to 0} T_D(a, \tau_0) = \frac{2\hbar}{\pi k_b\mathcal{T}}. \tag{15}$$

For large accelerations the temperature no longer depends on the lifetime,

$$\lim_{a\to +\infty} T_D(a, \tau_0) = T_U. \tag{16}$$

This is situation (2) which does not cause problems with respect to causality. For intermediate acceleration (9) is not satisfied for large $\tau_0$ (see fig. 1) so we have to check for (10).

[†] Notations are those of ref.[11]: the observer's proper time $\tau$ is measured from $-\tau_0$ to $\tau_0$.

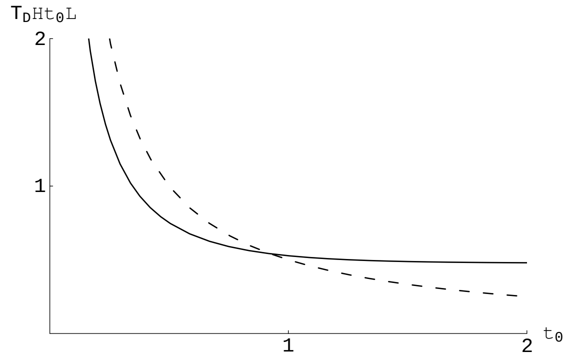


Figure 1: $T_D(\tau_0)$ for intermediate acceleration $a = 3$ (vertical axe in $\frac{\hbar}{k_B}$ unit). Causally acceptable points are under the dashed line (plot of $\frac{1}{k_B \mathcal{T}}$).

With respect to conditions (10) $T_D$ is always acceptable. Indeed

$$\Delta T(a, \tau_0) \doteq T_D(a, \tau_0) - T_U = \frac{\hbar}{\pi k_b \mathcal{T}} f(a\tau_0) \qquad (17)$$

with

$$f(x) \doteq x(\frac{\cosh x + 1}{\sinh x} - 1). \qquad (18)$$

Since

$$\lim_{x \to 0} f(x) = 2 \,, \lim_{x \to +\infty} f(x) = 0 \qquad (19)$$

and $f'$ is negative on $\mathbb{R}^{*+}$ then

$$0 \leq \Delta T(a, \tau_0) \leq \frac{2\hbar}{\pi k_b \mathcal{T}} \qquad (20)$$

which satisfies (10) for any lifetime $\mathcal{T}$.

# IV Conclusion

For a small acceleration, diamond's temperature $T_D$ cannot be distinguished from the ground energy of the detector, whatever the lifetime is. For intermediate and large accelerations $T_D$ cannot be distinguished from $T_U$. In all cases an Unruh observer is not able to deduce information on his lifetime from the measurement of the vacuum's temperature. Thus diamond's temperature is a causally acceptable adaptation of Unruh effect to the non eternal case. In this framework it might be interesting to re-evaluate the intermediate result (time-dependent) of ref.[14] that its author estimated as non physical.

One may find that the argument of this note - a non-eternal observer does not live long enough to realize that he is not eternal - is quite paradoxical. Moreover corrections to Unruh temperature seems of poor interest at first sight since they are causally acceptable only if they are not physically detectable. Such assertions lie in the identification between the observer and the quantum system interacting with the vacuum. More precisely one expects that at a given instant of its lifetime the quantum system delivers the instant temperature of the vacuum. A more plausible possibility is to expect the measurement to occur *after* the end of the system. For instance one may think of a particle process in which Unruh temperature, including corrections, corresponds to a correlation between the lifetime of the particles and their production (or disintegration) rate. Such process have been proposed[12], they give a concrete signification to Unruh temperature and seem more closed to experimental realization than abstract quantum system coupled to the vacuum.

Even assuming that the corrections to $T_U$ are not detectable (which seems plausible for $T_U$ itself is already extremely small) the good causal-behavior of $T_D$ validates the application of Unruh effect to a non-eternal observer. Since it is non zero even for an inertial observer, it suggests that the origin of the thermalization process lies more in the existence of an horizon than on the acceleration itself. This questions the treatment of a diamond-shape horizon in terms of (local) entropy, as this has been done for Rinder horizon[9].

Finally, note that time-energy uncertainty relation deals more with size orders than with exact values (in literature one often finds (5) with "$\gtrsim$" rather than "$\geq$"). Since (15) and (20) do satisfy (9) and (10) thanks to a factor $\frac{2}{\pi} \backsim 1$, it appears that diamond's temperature is the maximum value that one can canonically assign to a finite region of Minkovski spacetime.

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